Three Dimensional Finite-difference Time-domain Slotline Analysis on a Limited Memory Personal Computer

Qiang Chen and V. F. Fusco

Abstract— **In order to reduce computer memory and computational time required by three dimensional Finite-Difference Time-Domain(FDTD) microwave passive component analysis, two strategies are developed in this paper. First, the recently reported dispersive boundary condition (DBC) is modified thereby enabling the absorbing boundary to be located near to the main field area; second, an exact source plane field distribution itself produced by the FDTD algorithm is adopted to improve numerical accuracy. This leads to a reduction in computer memory requirements. As an example, a full-wave 3d FDTD analysis of a slotline is performed on a 33MHz PC486. The dispersion characteristic of the slotline is presented up to 1000 GHz this agrees well with that of closed form formula. When compared with previous slotline solutions, frequency range results are extended by an octave and less than 9 percent of the computer memory previously required is demanded by this paper. The unique and highly efficient combination of the two strategies presented here could be applied to other microwave waveguide component analyses.**

Index Terms—**Finite-difference time-domain, Slotline, Absorbing boundary conditions**

I. INTRODUCTION

THE three dimensional Finite-Difference Time-Domain (FDTD) method is a versatile method for the analysis of waveguide components and scattering problems. A wide frequency band characterisation of the structure can be obtained within one FDTD analysis. However the method can result in large computer memory and long computational time requirements. Usually a 3d FDTD analysis has to be performed on a work station, thus restricting the wider application of the 3d FDTD method. In addition, two reasons, apart from the algorithm itself, which result in limitation of use are the absence of high efficiency artificial absorbing boundary conditions and the difficulty of obtaining an accurate transverse field distribution on the source plane. Firstly , the artificial boundary conditions often used is Mur's first order condition (Mur's FOC) [1], which cannot absorb the incident wave effectively for structures with complex boundaries, slotline is an example. It is well known that small time domain errors can lead to huge frequency domain errors once the time domain results are Fourier transformed. So for inclusion of the side walls of a microwave structure researchers have to enlarge the analysis domain in order to allow the fields to decay before they reach the boundaries. This reduces the numerical errors caused by imperfect boundaries but is computationally expensive. Secondly, an accurate field distribution on the source plane is essential for exciting a distortion-free pulse along the transmission line. If the initial field distribution on the source plane is not matched to the line, there will be a serious amplitude distortion of the input pulse immediately after it is launched. This could make the whole analysis a failure. Therefore researchers currently tend to use a priori based quasi-TEM estimation of the field distribution on the source plane. Generally since this estimation is not accurate enough, the fields are required to travel a finite distance from the source plane in order to settle down to the transmission line mode. Once again this means extra computation domain and computational time requirements.

In this paper, the above drawbacks are overcome by applying the recently published dispersive boundary condition (DBC) [2] in a modified form in conjunction with a new approach which yields exact source plane field distributions which themselves are obtained from the FDTD algorithm. Mode templates have been shown by Railton and McGeehan [3] to improve the accuracy of the FDTD method in microstripline analysis. These two approaches when combined greatly reduce the computation domain and computational time and also enhance the results' accuracy dramatically. Consequently, a 3d FDTD analysis can now be performed on a personal computer. This should greatly enlarge the application area of the 3d FDTD method and make the method more acceptable to both microwave researchers and engineers. In this paper, as an example, a slotline is analyzed. The field distributions both on the source plane and along the slot are presented to show the validity of the method. The computed wide frequency range dispersion characteristic of the slotline is compared with closed form formula [4] and good agreement is achieved. Due to the considerable improvement obtained by applying the modified DBC and exact source plane field distributions, the computer memory requirements of this paper are less than 9 percent of those in [5].

II. FIELD DISTRIBUTION ON SOURCE PLANE

For a multi-dielectric open air transmission line, accurate field distributions on a transverse plane are very difficult to obtain from static field analysis. As mentioned above, in order

Manuscript received August 26, 1993; revised April 16, 1994. This work was supported by a Queen's University Belfast visiting studentship.

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IEEE, Log Number 9407288.

to excite a distortion-free Gaussian pulse, exact transverse field distributions should be computed first and then applied on the source plane. In this paper the FDTD algorithm itself is utilized to generate these field distributions. Fig. 1 shows the slotline configuration. First, an initial estimate of the E_x field in a transverse plane of the slotline is calculated by assuming uniformly distributed electrical charges on the perfect metal strip, Fig. 2. By integrating the electrical field produced by each electrical charge, we obtain

$$E_x = -\int_{-\infty}^{x_1} \frac{x_s - x_0}{[(x_0 - x_s)^2 + (z_0 - z_S)^2]^{3/2}} dx_s + \int_{x_2}^{\infty} \frac{x_s - x_0}{[(x_0 - x_s)^2 + (z_0 - z_s)^2]^{3/2}} dx_s \qquad (1)$$

where on the surface of the metal strip $E_x = 0$. Since only the relative field strength is of interest, the constant coefficient $q/(4\pi\varepsilon)$ is omitted here. The computed value of E_x calculated according to (1) satisfies the boundary condition at the interface of air and dielectric substrate since E_x is a tangential field component there. However E_z is difficult to calculate directly because the vertical boundary condition must be satisfied at the interface of air and substrate. The computed value of E_x is applied at every grid in the transverse plane a few space steps (3 steps are adopted in this work) from the $y = 0$ plane. Absorbing boundary conditions have been applied at both end planes, $y = 0$ and $y = L$. Next in order to excite steady field distributions in the slotline by applying a FDTD algorithm, we assume E_x has the time dependence

$$E_x(t) = \frac{e^{0.1(t-t_0)}}{1 + e^{0.1(t-t_0)}} \qquad (2)$$

where t_0 decides when this dependence goes from zero to one and the coefficient 0.1 smooths the transition. In order to establish the field smoothly, t_0 should be sufficiently large. In this work t_0 is selected to be 70, Fig. 3. Once the steady state is achieved (in this paper's example 400 time steps are required, see Fig. 3), any of the six field components on a transverse plane of the slotline can be accurately obtained. The field components E_x and E_z are sampled at $y = 14$ of the total 20 space steps used for the y direction. Fig. 4(a) and Fig. 4(b) show the E_x and E_z components respectively, these give a direct view of the transverse field distributions. Notice that because of symmetry, only the left half of the slotline is analyzed here. Fig. 5(a) shows a Gaussian pulse launched from the source plane travelling down to the end of the slotline where it is absorbed by the boundary, i.e. a plot of E_x component in the centre of the slot at different y positions. From Fig. 5(a) we can see that there is no sharp fall in the amplitude of the pulse near the source plane after the pulse is launched. This means the source plane fields are perfectly matched to the slotline so that there is no energy lost to the pulse there.

The above procedure is carried out only for part of the total computation domain (here only 20 space steps in y direction are used), i.e. a very short piece of slotline, leading to only a small time overhead. After the initial field distributions on the source plane are obtained, this computation domain is released for waveguide dispersion characteristic analysis. This means no extra memory is required by the procedure. Since accurately computed transverse field components E_x and E_z are used on the source plane, no extra length of slotline is required in order for the excitation field to settle down to its transmission line mode. Therefore the reference plane for the dispersion characteristic analysis in this paper can be made the same as the source plane.

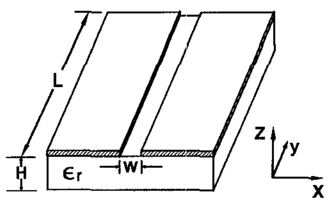

Fig.1. Slotline configuration

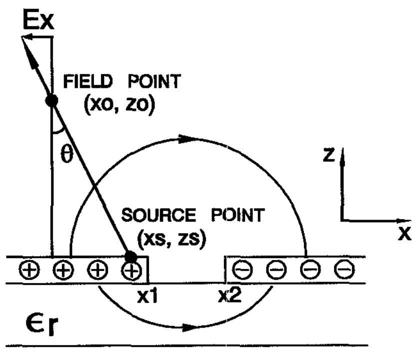

Fig. 2. Static field analysis on a transverse plane of slotline

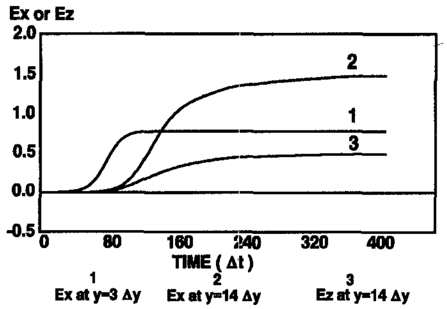

Fig. 3. Steady fields establishment along a short piece of slotline

III. Modified Dispersive Boundary Condition

The often used artificial boundary condition is Mur's FOC [1]. It is simple to use and has reasonable accuracy for low dispersion microwave structures. But theoretically it can only absorb one frequency component completely. For other frequencies, there will be increased or decreased reflection depending upon the frequency offsets. A DBC is virtually the multiplication of two items of Mur's FOC, where each item is constructed to absorb a different frequency component. Therefore, the multiplication has a lower reflection coefficient than each separate one over a wide frequency range. At the x

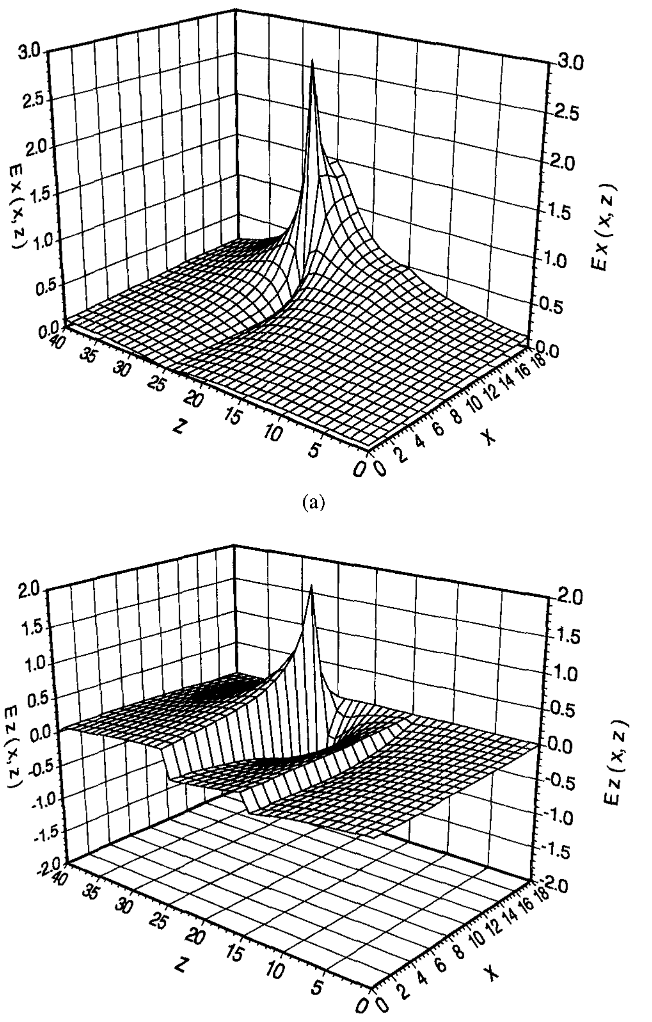

Fig. 4. Field distribution on a transverse plane of slotline. (a) E_x; (b) E_z. (X : 0 to 15, left metal strip; 15 to 18, left half slot; Z : 0 to 15, air; 15 to 25, dielectric substrate; 25, metal strip and slot; 25 to 40, air.)

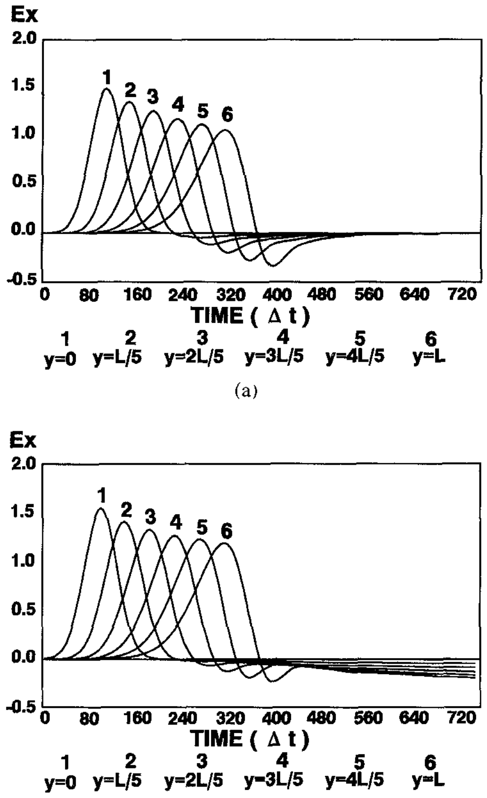

Fig. 5. A Gaussian pulse propagates along the slotline under different absorbing boundary conditions. (a) modified DBC at side wall boundaries, $\alpha_{x1} = 0.965$, $\alpha_{x2} = 0.98$, $\alpha_{y1} = 1$, $\alpha_{y2} = 1$, $\alpha_{z1} = 0.95$, $\alpha_{z2} = 1$; $\varepsilon_{reffx1} = 3$, $\varepsilon_{reffx2} = 10$, $\varepsilon_{reffy1} = 7$, $\varepsilon_{reffy2} = 8.5$, $\varepsilon_{reffz1} = 3$, $\varepsilon_{reffz2} = 10$. (b) conventional DBC with the same effective dielectric constants as modified DBC's.

direction left boundary, the DBC for the E_z or E_y component can be expressed as

$$\left(\frac{\partial}{\partial x} - \frac{1}{v_1}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} - \frac{1}{v_2}\frac{\partial}{\partial t}\right)E = 0 \tag{3}$$

where travelling wave velocities v_1 and v_2 are correspondent to two different frequencies. However, in this paper's application of the DBC in the slotline case, we find that when the side wall boundary is near the slot, the DBC is barely stable and has a DC offset. Fig. 5(b) shows a Gaussian pulse travelling dispersively along the slot under DBC, a DC offset of the tails is obvious. In order to alleviate this problem, we need to reconsider each of Mur's FOC items in (3).

In Mur's FOC, the following assumption is made

$$[1-(C_0S_y)^2-(C_0S_z)^2]^{1/2} \approx 1+O[(C_0S_y)^2+(C_0S_z)^2] \approx 1 \tag{4}$$

i.e. $O[(C_0S_y)^2-(C_0S_z)^2]$ is assumed to be nil. This means a perfect plane wave is assumed and this is not accurate enough for the work presented here. An improvement could be made as a second order approximation, but this makes the dispersive boundary condition very complex. In this work, instead of nil in Mur's FOC, we denote $[1\text{-}(C_0S_y)^2\text{-}(C_0S_z)^2]^{1/2}$ as $1+O_1$. Here O_1 should be numerically very small and is a function of time and space [6]. We make a first approximation of O_1 , i.e. $O_1 = \eta t + \xi$, where η and ξ are functions of space. At the boundary, η and ξ can be treated as constants since x is fixed there. So the first order absorbing boundary condition can be recast as

$$\left[\frac{\partial}{\partial x} - \frac{1}{C_0}\frac{\partial}{\partial t}(1+\eta t+\xi)\right]E = 0 \tag{5}$$

Equation (5) can be written as

$$\left[\frac{\partial}{\partial x} - \frac{1+O_1}{C_0}\frac{\partial}{\partial t} - \frac{\eta}{C_0}\right]E = 0 \tag{6}$$

From (6) we can see that the introduction of O_1 in coefficient $(1+O_1)/C_0$ is in fact the velocity corresponding to an effective dielectric constant ϵ_{reff} in the dispersive boundary condition and can be denoted as v, i.e. $v = C_0/(\epsilon_{reff})^{1/2}$. Also if we denote "$\eta/C_0$" as "$\tau$", then the new Mur's FOC is

$$\left[\frac{\partial}{\partial x} - \frac{1}{v}\frac{\partial}{\partial t} - \tau\right]E = 0 \tag{7}$$

Replacing each item in (3) with (7), we obtain

$$[E_0^n - \alpha_1(E_1^{n-1} + \gamma_1 E_0^{n-1} - \gamma_1 E_1^n)] \cdot [E_0^n - \alpha_2(E_1^{n-1} + \gamma_2 E_0^{n-1} - \gamma_2 E_1^n)] = 0 \quad (8)$$

where

$$\alpha_i = \left(1 + \tau \frac{2v_i \Delta t \Delta x}{\Delta x + v_i \Delta t}\right)^{-1} \quad \gamma_i = \frac{\Delta x - v_i \Delta t}{\Delta x + v_i \Delta t} \quad (9)$$

and E_M^n represents the nth time-step of the tangential electrical field component at spatial position M nodes inside the boundary. From (9) we can see that instead of selecting τ, we now select α_i which should have a value of approximately 1 since τ_i is relatively small. When both α_1 and α_2 are equal to 1, (8) becomes conventional DBC. Expanding and rearranging (8), we obtain

$$\begin{aligned} E_0^n = &(\alpha_1 + \alpha_2)E_1^{n-1} - \alpha_1\alpha_2 E_2^{n-2} + (\beta_1 + \beta_2)(E_0^{n-1} - E_1^n) \\ &+ (\beta_1\alpha_2 + \beta_2\alpha_1)(E_2^{n-1} - E_1^{n-2}) \\ &- \beta_1\beta_2(E_0^{n-2} - 2E_1^{n-1} + E_2^n) \end{aligned} \quad (10)$$

where

$$\beta_i = \alpha_i \gamma_i \quad (11)$$

In order to obtain a distortion-free transmitted Gaussian pulse, the coefficients ϵ_{reffi} (corresponding to v_i) and α_i in the modified dispersive boundary condition should be carefully selected. We have found ϵ_{reffi} at boundary $y = L$ is more effective than those at $z = 0, z = Z_{max}$ (Z_{max} is 40 space steps in this work) and $x = 0$, because the fields there are much more stronger than those at any other boundaries. Careful selection of ϵ_{reffi} can minimize the reflected ripples in the tail of the Gaussian pulse as discussed in [2]. In this work ϵ_{reffi} are determined as $\varepsilon_{reffx1} = 3$, $\varepsilon_{reffx2} = 10$, $\varepsilon_{reffy1} = 7$, $\varepsilon_{reffy2} = 8.5$, $\varepsilon_{reffz1} = 3$ and $\varepsilon_{reffz2} = 10$. The α_i coefficients are selected to remove the DC offset. The criteria is to ensure the tail of Gaussian pulse tends to zero when the simulation time is sufficiently long. For the slotline example discussed in this paper we find that the α_i coefficients should be selected less than or equal to 1, otherwise the artificial boundary conditions may become unstable. The values of α_i used are as $\alpha_{x1} = 0.965$, $\alpha_{x2} = 0.98$, $\alpha_{z1} = 0.95$ and $\alpha_{z2} = 1$. At the boundary $y = L$, since the fields there are more plane wave like than those at other boundaries, the α_i values are selected to be 1. Fig. 5(a) shows a travelling Gaussian pulse along the slot where the above modified DBC is applied to the side wall boundary. Fig. 5(a) indicates much less distortion than Fig. 5(b).

IV. Dispersion Characteristic

From Fig. 5(a), we take the Fourier transform at the front end (source plane position in this paper) and at the back end of the line, from these the dispersion characteristic of the slotline can be obtained in the conventional manner. In order to compare the computation efficiency with [5], the same size slotline is analyzed, where the thickness of the substrate is $H = 0.1$mm, the slot width is $W = 0.06$mm, and the dielectric constant of the substrate is ε_r=13. From Fig. 6 we can see this paper's curve agrees well in an extreme wide frequency range with that of closed form formula based on Cohn's analysis [4] which means very accurate results are obtained by the method developed in this paper. Thus the combined use of high efficiency dispersive boundary conditions developed in this paper and exact field distributions on the source plane, mean that the computation domain is greatly reduced. In this paper, a box with sides $(18\Delta x)\times 40(\Delta y)\times 40(\Delta z)$ was used, where $\Delta x = \Delta y = \Delta z = 0.01$mm. Compared with [5] which uses a box of $(55\Delta x)\times(100\Delta y)\times(60\Delta z)$, the computation domain of this paper is less than 9 percent of that required there. Consequently the computational time is only 18 minutes on a 33MHz PC486.

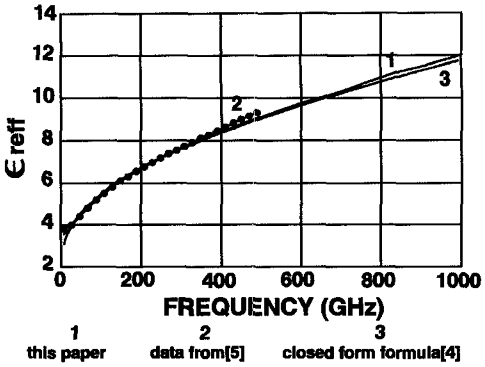

Fig. 6. Comparison of the effective relative dielectric constant for slotline

V. Conclusion

A modified dispersive boundary condition in conjunction with exact source plane field distributions' themselves produced by the FDTD algorithm result in a great reduction of both computer memory and computational time. The computed dispersion characteristic of a slotline by this paper's method agrees very well with closed form formula in a wide frequency range. The strategy developed in this paper could be applied to other open waveguide structures.

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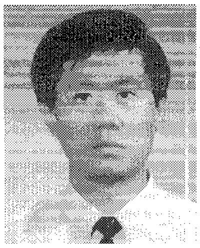

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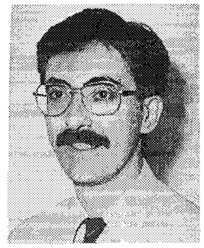

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